ROBUST AND AUTOMATIC VANISHING POINTS DETECTION WITH THEIR UNCERTAINTIES FROM A SINGLE UNCALIBRATED IMAGE, BY PLANES EXTRACTION ON THE UNIT SPHERE

Mahzad Kalantari [a,b,c], Franck Jung[d], Nicolas Paparoditis[a] and Jean-Pierre Guedon[b,c]

[a]MATIS Laboratory, Institut Geographique National
2, Avenue Pasteur. 94165 SAINT-MANDE Cedex - FRANCE - firstname.lastname@ign.fr
[b] Institut Recherche Communications Cyberntique de Nantes (IRCCyN) UMR CNRS 6597
1, rue de la No BP 92101F-44321 Nantes Cedex 03 - FRANCE - firstname.lastname@polytech.univ-nantes.fr
[c] Institut de Recherche sur les Sciences et Techniques de la Ville CNRS FR 2488
[d] DDE - Seine Maritime - firstname.lastname@equipement.gouv.fr

ABSTRACT:

This paper deals with the retrieval of vanishing points in uncalibrated images. Many authors did work on that subject in the computer vision field because the vanishing point represents a major information. In our case, starting with this information gives the orientation of the images at the time of the acquisition or the classification of the different directions of parallel lines from an unique view. The goal of this paper is to propose a simple and robust geometry embedded into a larger frame of image work starting with an efficient vanishing point extraction without any prior information about the scene and any knowledge of intrinsic parameters of the optics used.After this fully automatic classification of all segments belonging to the same vanishing point, the error analysis of the vanishing points found gives the covariance matrix on the vanishing point and on the orientation angles of the camera, when using the fact that the 3D directions of lines corresponding to the vanishing points are horizontal or vertical. A validation of estimated parameters with the help of the photo-theodolite has been experimented that demonstrate the interest of the method for real case. The algorithm has been tested on the database of a set of 100 images available on line.

1 INTRODUCTION

In the conical geometry characteristic of human vision, or of photography, the parallel lines in the object space result in the image as pencils that intersect on vanishing points. Therefore to every vanishing point, a 3D direction is associated. The intersection of the lines in the image, corresponding to a given vanishing point, is mathematically an ill-conditioned problem. The various lines intersect with very small angles, which badly conditions the classic model of intersection. Another problem is a problem of classification, namely which segment is associated to which vanishing point. Since the years 80 many authors have tried to provide solutions to these two difficulties. In order to summarize the difference between all these approaches, one could say that it resides in the differences of parameter spaces for modelling and resolving the problem. Several types of spaces for modelling exist. One of the most widely known is the accumulation space on the Gauss sphere, that has been introduced by Barnard (Barnard, 1983) and that since then has been used, modified, and optimized by many different authors (Magee and Aggarwal, 1984)(Shufelt, 1999). Other authors used other accumulation spaces using the Hough transform (Lutton et al., 1994)(Quan and Mohr, 1989)(Tuytelaars et al., 1998). The modelling can also be performed in the space object, which was used by many authors (Antone and Teller, 2000). Then must be mentioned all authors who have modelled the problem in the 2D space of the image (van den Heuvel, 1998)(Brauer Burchardt and Voss, 2000)(Schaffalitzky and Zisserman, 2000)(Rother, 2002). In general this kind of approach requires the use of probabilistic models (Almansa et al., 2003). Most methods of extraction of vanishing points are composed of 2 steps, one for the vanishing points detection, and another for the association of the different directions of the segments in the image with these vanishing points. The method presented in this paper has for parameter space an unit sphere without accumulation process. It works in only one step, so that the extraction is made simultaneously with the classification. The details of the geometry framework used in the method is described in the section 2. The different steps of the algorithm are described in the section 3. In order to have an assessment of the uncertainty on the location of the vanishing point, a very detailed covariance analysis has been done, and is the topics of the section 4. The section 5 illustrates the results of classification directly on some images taken as examples. Finally, in order to compare with a ground truth, a precise measure instrument that can give the orientations of the image acquisition was necessary, and thus a photo-theodolite (camera mounted with a precise goniometer) has been used.

2 GEOMETRIC FRAMEWORK

In this section, is presented the geometry on which the algorithm is based. The origin O is chosen at a distance D from the image plane. The axes Y and Z are parallel to the axes of the image, and thus the X axis is perpendicular to the image plane.

l_1 and l_2 are the extremities of a given segment in the image. If this segment belongs to a family of segments who converges on the vanishing point V, one is interested in the corresponding family of planes of the space who pass through O, each containing one of these segments. V being on the line carrying the segment, a given plane contains therefore always V, and therefore the line OV too. For each of this family's planes, the normal vectors passing through O are all coplanar, because they are included in the plane passing by O and perpendicular to OV (Figure 1). And the normal in O to this last plane, (that we call "vanishing plane" in the present paper for this reason), intersects the image plane in the vanishing point V. Thus the extraction of V may be obtained through the research of the best plane containing all the extremities of normal vectors to the previously defined planes, each of them containing O, V and a given segment. It is important to note
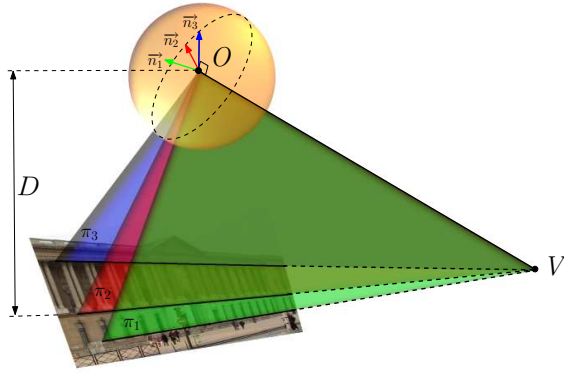
Figure 1: Geometry of the image and the vanishing plane, whose normal in O intersects the image on the vanishing point V

that if O is chosen at the optic centre, and therefore if D is the focal length F, then the plane previously defined is the one that, in the literature of computer vision, is often called the interpretation plane (Barnard, 1983) (Weiss et al., 1990)(Antone and Teller, 2000). But the present work doesn't stand in this very particular situation, which requires a previous knowledge of the intrinsec parameters of the camera. Here the point O is taken in an arbi-
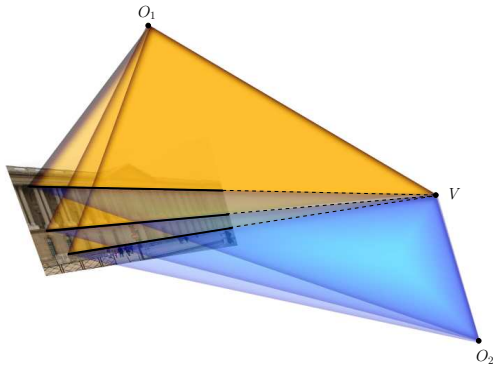

Figure 2: Illustration of the geometric features used in the algorithms : the set of planes built on the segments of the image and containing O intersect in V, whatever the position of O.

trary way (Figure 2), obviously outside of the image plane, and for simple reasons of numeric stability, at a reasonable distance from the centre of the image. This remark is important, because the algorithms presented here require no prior knowledge of the optics used to extract correctly the points V. It works in the same way as the eye does, when it looks at an image, and rebuilds the vanishing points mentally without knowing the elements of the optics used to get this image.
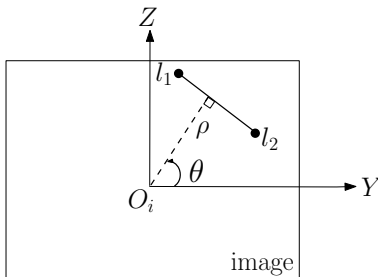

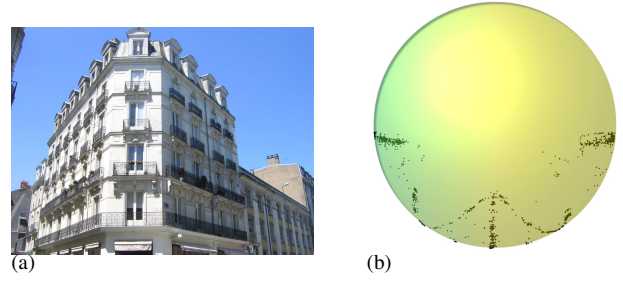Figure 3: Definition of the reference system on the image

Figure 4: Typical urban scene (a). On (b), the representation of the corresponding ends of vectors, normal to the planes including each segment and the origin O : one clearly sees the clouds of points, approximately along great circles on the unit sphere centered on O. There are as many circles as vanishing points.

$$X \sin \theta + Y \cos \theta = \rho. \tag{1}$$

N is the vector, passing through O, normal to the plane formed by l_1, l_2 and O. Each line of the ZY plane is defined by its polar equation (equation 1) as a function of ρ and θ, polar coordinates of every segment in the image plane (using the origin of the image) (Figure 3). The normal vector to this plane can be calculated directly from ρ and θ. The extremities of all these normalized normal vectors lie therefore on a unit sphere (equation 2).

$$N_x = -\frac{\rho}{\sqrt{D^2 + \rho^2}} \quad N_y = -\frac{D \sin \theta}{\sqrt{D^2 + \rho^2}} \quad N_z = -\frac{D \cos \theta}{\sqrt{D^2 + \rho^2}} \tag{2}$$

That way, a mapping on this unit sphere has been performed (Figure 4). Some authors have used this geometry. But all of them took their origins on the optic centre, therefore at a distance F of the image, which requires the knowledge of the intrinsic parameters. Here, the problem of detection of the different families of directions of the image corresponding to parallel directions of the object space, is converted into the detection of all meaningful planes formed by the extremities of the normal vectors issued from O. In the next parts of this paper the extraction method of the planes is deepened. It is completed by an error analysis on the results obtained for the vanishing point.

3 ALGORITHM OF EXTRACTION OF THE VANISHING PLANES

Four important points of this algorithm can be mentioned:
1 - there is no need to make any hypotheses on the number of vanishing points present in the image,
2 - the algorithm is robust and use levels that adapt to the precision of detection of the segments,
3 - all vanishing points are detected; there is possibly an over-detection, but no under-detection,
4 - no prior information on the optics intrinsec parameters is requested.
The basic algorithm for the detection of the vanishing points requires two steps, followed if needed by an error analysis of the vanishing point.
The main steps are:
1 - Segments detection in image,
2 - Calculation of the normal vectors and extraction of the vanishing planes,
3 - Analysis and error propagation on the vector of vanishing point.

3.1 Detection of segments in the image

As it is the case for every algorithm of extraction of vanishing points, the first step is the detection of the image segments. Many algorithms of detection of segments are available (Burns et al., 1987)(Deriche, 1987)(Deriche et al., 1992). In the proposed method the detection of the segments is based on the Deriche Vaillant algorithm (Deriche et al., 1992). The advantage of this approach is that it can produce for every segment a covariance matrix for the line carrying this segment. The equation of this line is expressed in polar coordinates (ρ and θ) and a covariance matrix is provided on these parameters. But any other segments detector may also be chosen.

3.2 Calculation of the normal vectors and extraction of the vanishing planes

Once the segments detected on the image as well as the lines carrying these segments, the normal vectors can be easily calculated (equation 2). Now the problem lies in the extraction of the different planes in a set of clouds of points. For overcoming this problem, a method inspired of the RanSac(Fischler and Bolles, 1987) was implemented. RanSac is a robust estimator based on a random sample principle. All necessary parameters for the use of the RanSac are described briefly in (Fischler and Bolles, 1987), this article being adapted for an unique cloud of points. But in the case of the vanishing planes, several clouds of points coexist. It is therefore necessary to bring some changes to the RanSac method. The main changes concern :
- The error tolerance for establishing datum/model compatibility t (Algorithm 1)
- The maximum number of attempts to find a consensus set m
- The lower bound on the size of an acceptable consensus set T
The principle of the algorithm is the following one:

1. - 2 normal vectors are randomly selected (N_1, N_2)

2. - Calculation of the vectorial product between the 2 normal vectors, the vectorial product of N_1 with N_2 defines the normal to the plane (V_p)

3. - The whole set of normal vectors is browsed to find other normals coplanar with the two first ones, with the condition that V_p is orthogonal to them with a tolerance t (Algorithm 1)

4. - After m iterations the plane that aggregated the biggest number of normal vectors is kept as a vanishing plane

5. - The normal vectors classified in a plane are withdrawn from the main set

6. - The whole previous process is repeated until there are only 4 normal vectors left in the set. So that here, $T = 4$

7. - In each set of normal vectors selected, the best plane is computed by a classical least-squares adjustment.

The number of samples cannot be calculated conventionally with the probabilistic method given by Fischler and R. C. Bolles (Fischler and Bolles, 1987) because no prior knowledge is available. It is impossible to say in advance what percentage of points is good and what percentage is not. A normal vector belonging to a plane is a "inlier" for its plane and will be considered an outlier for another plane, it is therefore impossible to decide on a strict probabilistic basis. In an empiric way, it has been found satisfactory to take for m the half of the number of segments of the image. So the choice of m adapts himself to the image, and therefore the process remains entirely automatic.

Algorithm 1 Calculate tolerance (t) for the RanSac
___
$Sum \Leftarrow 0$
for $i = 0$ to *size of the set of normal vectors* **do**
  Calculate covariance matrix of N deduced from covariance matrix of θ and ρ
  $\sum_{N_{x,y,z}} = J \sum_{\theta,\rho} J^t$ {J is the matrix of partial derivatives of N (equation 2) with respect to θ and ρ}
  $\overrightarrow{\delta N} = \overrightarrow{N} + (\sigma_{N_x}, \sigma_{N_y}, \sigma_{N_z})$
  $\alpha_i = |\arccos\left(\frac{(N|\delta N)}{||N||\cdot||\delta N||}\right)|$

  $Sum \Leftarrow \alpha_i + Sum$
end for
Calculate tolerance (t)
return $t \Leftarrow$ *median of Sum*
___

4 ERROR PROPAGATION ON THE VANISHING POINT

After the step of the classification of the planes, it is important to be able to quantify, by a covariance analysis, the uncertainty of the vector of the vanishing point. The impact of the uncertainty of the normal vectors calculated on the vector of the vanishing point is thus computed. The normal vectors are the observations. It is therefore necessary to adjust the best plane fitting these normal vectors while taking in account their uncertainties. The process of least squares used here, using the covariance matrix of the observations, is the method of Helmert-Gauss ((Helmert, 1872)(Cooper, 1987)). It is supposed that all error on the normal vectors (N) have a Gaussian distribution. The cost function γ here is the square of the distance between the extremities of the normal vectors and the plane that passes through the origin:

$$(aN_x + bN_y + cN_z)^2 - d^2 = \gamma. \qquad (3)$$

a, b, c are the components of the extremity of each vector of the normal to the plane. The aim is to estimate and to calculate the covariance matrix on a, b and c. The method of Helmert Gauss is formulated here in the following way:

$$Ax + Bv = b. \qquad (4)$$

A is the Jacobian matrix of γ with respect to the unknown parameters (a, b and c), B is the Jacobian matrix of γ with respect to the unknown observations (N_x, N_y and N_z). The resolution of this system is performed by the minimization of $v^t W v$, v being the vector of the observations. The resolution of the system uses the multiplier of Lagrange in an iterative way. For each iteration the vectors x and v are calculated. The convergence is obtained when the values don't evolve beyond 1 pixel. Once obtained the covariance matrix on the vector of the calculated vanishing point, one propagates it on its intersection with the image plane. Thus an assessment of the uncertainty on the localization of the vanishing point in the image plane is got.

5 RESULTS AND ASSESSMENT

In order to show some examples of results of the classification of the different directions of lines in an image, 2 examples in the images data base have been chosen. The first image is the Louvre pyramid in Paris (Figure 5), chosen as it contains a lot of families of directions of parallel lines. In this image the vectors of vanishing points have the same colour as the extremities of normal vectors belonging to the same plane. A second image (Figure 6) shows that the algorithm is robust and its parameters tune themselves automatically with regard to the quality of detection of the segments.
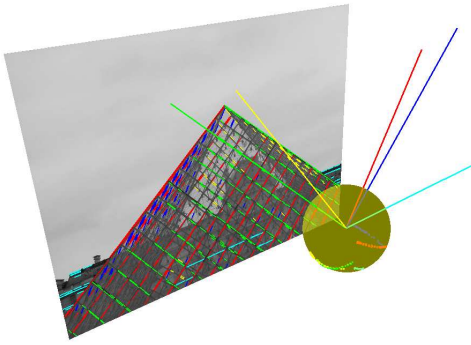
Figure 5: Typical result on an image of the Louvre pyramide, Paris. The different families of parallel lines are displayed along with the end of the extremities of normal vectors issued from O relative to each corresponding segment, with the same colour.

Figure 6: Automatic classification of segments.

The possible methods of assessment for the algorithms of automatic vanishing points extraction are to be sorted. One can certainly value their efficiency in terms of numbers of vanishing points found, but no reference method exists to find the correct value except by visual manner. Besides, to find a conclusive statistical value is not that obvious: it is quite simple to bias the results, either by doing the tests on images of neighbouring geometries (which induces a bias on the random character of the sample), either by choosing too simple images (all results are then favourable), or unusually complex, such of unclassical buildings where all lines are curves (and the results will be abnormally bad). In the present case, the assessment was based on about one hundred relatively varied images (Figure 7), for which the number of vanishing points found (ranging from 0 to 5 according to the visual inspection of images) was satisfactory. An important set of images, which served to deepened tests, has been put on line on the site http://mahzad.kalantari.free.fr/pdf.htm. One can also try to measure in an independent and more precise way the coordinates that one should find for the vanishing points, and then two options are possible: to work on synthesis images, or to use a photo-theodolite. This last one permits to get images with a calibrated camera, and the orientation of the optical axis is measured with an extreme precision thanks to the theodolite that is included in the mount of the camera. The validation has been led in the following way : eight successive images have been taken of a given scene, presenting a building whose three vanishing points corre-
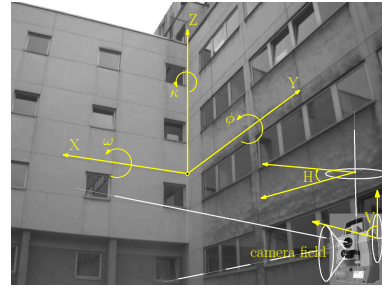

Figure 7: Exemple of image extrated from the database

Figure 8: Definition of the angles ω, ϕ and κ of the object space, showing the relations with the angles H and V of the photo-theodolite.

spond to orthogonal directions, with different angles of view. The angles of the photo-theodolite have been recorded every time. The eight images have been processed in order to extract the vanishing points in an automatic way. The angles ϕ and κ, computed from (Patias and Petsa, 1993) using the values of focal length and position of the optic center as results of a calibration, are bound directly to the angles H and V measured on the theodolite. Indeed, unlike the situation that prevails for the automatic recovery of vanishing points using the present algorithms, it is necessary, when exploiting the vanishing points in order to compute these angles, to use the intrinsic parameters. The three angles computed for each image follow the classical denomination in photogrammetry (ω, ϕ and κ), and their geometrical significance is presented in the Figure 8. The discrepancies of orientation between the successive images have been compared depending on whether one exploits the vanishing points or the values measured by the theodolite (considered as a reference). The standard deviations for the discrepancies on the angles ϕ and κ have been obtained on a set of eight images of the same scene under different angles of view (Table 1). The value obtained is 0.2 mrd for the component ϕ, that corresponds to the planimetric orientation of the facades observed. It is equivalent, to a distance around 10 m (mean distance from the camera to the facades) to an error of two mm, which is therefore a satisfactory order of magnitude. One deduces that the algorithm works with a good precision on this type of scene. The discrepancies are higher, around 10 mrd, with the vertical angles extracted from the vanishing points κ, as compared with H angles. This is due to the high uncertainty on the intersection of the images of the vertical lines of the object, the consequence being to provide a comparably poor value for κ.

σ_ϕ(mrd)	σ_κ(mrd)
0.2	10

Table 1: Standard deviation of ϕ and κ

6 CONCLUSION

A new algorithm, based on very simple geometric considerations and strongly helped by a very efficient RanSac-type methodology of classification, allows for a fully automatic extraction of the vanishing points available in a given image. This method is fully independent of any preliminary knowledge of the intrinsec parameters of the optics used, which facilitates its use in a wide set of practical situations. The computation time required for classical urban scenes ranges around one second with a classical PC, which is adapted to most possible applications. On another hand, a new methodology for testing the algorithms has also been developed, based upon the use of a photo-theodolite. This has provided a very efficient way to check their accuracy, which appears as satisfying. Generally speaking, this method may find

some vanishing points in excess (artifacts of lines that intersect without being the images of parallel lines), but at least finds all the true vanishing points.

REFERENCES

Almansa, A., Desolneux, A. and Vamech, S., 2003. Vanishing point detection without any a priori information. IEEE Trans. Pattern Anal. Mach. Intell. 25(4), pp. 502–507.

Antone, M. and Teller, S., 2000. Automatic recovery of relative camera rotations for urban scenes. Vol. 02, IEEE Computer Society, Los Alamitos, CA, USA, pp. 282–289.

Barnard, S. T., 1983. Interpreting perspective images. Artificial Intelligence 21, pp. 435–462.

Brauer Burchardt, C. and Voss, K., 2000. Robust vanishing point determination in noisy images. In: International Conference on Pattern Recognition, Vol. 1, pp. 559–562.

Burns, J. B., Hanson, A. R. and Riseman, E. M., 1987. Extracting straight lines. In: M. A. Fischler and O. Firschein (eds), Readings in Computer Vision: Issues, Problems, Principles, and Paradigms, Kaufmann, Los Altos, CA., pp. 180–183.

Cooper, M. A. R., 1987. Control surveys in civil engineering. Nichols Pub Co.

Deriche, R., 1987. Using canny's criteria to derive an optimal edge detector recursively implemented. Int. J. Computer Vision 2, pp. 15–20.

Deriche, R., Vaillant, R. and Faugeras, O., 1992. From noisy edges points to 3d reconstruction of a scene : A robust approach and its uncertainty analysis. World Scientific Series in Machine Perception and Artificial Intelligence 2, pp. 71–79.

Fischler, M. A. and Bolles, R. C., 1987. Random sample consensus: A paradigm for model fitting with applications to image analysis and automated cartography. In: M. A. Fischler and O. Firschein (eds), Readings in Computer Vision: Issues, Problems, Principles, and Paradigms, Kaufmann, Los Altos, CA., pp. 726–740.

Helmert, F. R., 1872. Die Ausgleichsrechnung nach der Methode der kleinsten Quadrate. Teubner, Leipzig.

Lutton, E., Maitre, H. and Lopez-Krahe, J., 1994. Contribution to the determination of vanishing points using hough transform. IEEE Trans. Pattern Anal. Mach. Intell.

Magee, M. J. and Aggarwal, J. K., 1984. Determining vanishing points from perspective images. Computer Vision, Graphics, and Image Processing 26(2), pp. 256–267.

Patias, K. G. P. and Petsa, E., 1993. Experiences with rectification of non-metric digital images when ground control is not available. In: Proceedings of the XV International Symposium CIPA 2005, Bucharest.

Quan, L. and Mohr, R., 1989. Determining perspective structures using hierarchical hough transform. Pattern Recognition Letters 9(4), pp. 279–286.

Rother, C., 2002. A new approach for vanishing point detection in architectural environments. In: Proceedings of the British Machine Vision Conference (BMVC), Vol. 20.

Schaffalitzky, F. and Zisserman, A., 2000. Planar grouping for automatic detection of vanishing lines and points. Image and Vision Computing 18, pp. 647–658.

Shufelt, J. A., 1999. Performance evaluation and analysis of vanishing point detection techniques. IEEE transactions PAMI 21(3), pp. 282–288.

Tuytelaars, T., Gool, L. J. V., Proesmans, M. and Moons, T., 1998. A cascaded hough transform as an aid in aerial image interpretation. In: International Conference on Computer Vision, pp. 67–72.

van den Heuvel, F. A., 1998. Vanishing point detection for architectural photogrammetry. International Archives of Photogrammetry and Remote Sensing 32(5), pp. 652–659.

Weiss, R. S., Nakatani, H. and Riseman, E. M., 1990. An error analysis for surface orientation from vanishing points. IEEE Trans. Pattern Anal. Mach. Intell. 12(11), pp. 1179–1185.